SOLUTIONS TO PROF. BORCHERDS' 2005 FINAL

PEYAM RYAN TABRIZIAN

(1) Looks like a 'bumpy' function
(2) 108
(3) Use IVT, $f(x) = x^4 + 1 - 3x$, $f(0) = 1 > 0$, $f(1) = -1 < 0$
(4) $\frac{e^x(x+1)-e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$
(5) $-\sin(\cos(\cos(x)))\cdot(-\sin(\cos(x)))\cdot(-\sin(x)) = -\sin(\cos(\cos(x)))\cdot(\sin(\cos(x)))\cdot(\sin(x))$
(6) $y' = \frac{2-xy-y^2}{x^2+2xy}$
(7) $2^{57}\cos(2x)$
(8) Use MVT, $\frac{f(5)-f(1)}{4} \geq -1$, so $f(5) \geq 6$
(9) 1
(10) Sketching-exercise
(11) $x = 5, y = -5$
(12) $\frac{25}{12}$
(13) $f(x) = \frac{x^4}{4} - \frac{5}{4}$
(14) $f(x) = -\sin(x) + x + 1$
(15) 14
(16) -2
(17) $3 + \frac{9}{4}\pi$
(18) Use the fact that e^{-x} is decreasing on $[0, 1]$, so $\frac{1}{e} \leq e^{-x} \leq 1$
(19) e^{-x^2}
(20) $\cos(x)\tan(\sin(x)) + \sin(x)\tan(\cos(x))$
(21) 2
(22) $\sqrt{2} - 1$
(23) $\frac{1}{2}\left(1 + y^2\right)^{11}$
(24) $\frac{-1}{2}\left(\ln(\cos(x))\right)^2$
(25) $\frac{1}{4}$
(26) Let $f(x) = \frac{1}{x}$ and basically $\int_1^n f(x)dx$ is smaller than the left-hand-sum L_n
(27) $\pi - \frac{2}{3}$
(28) 40π
(29) $\frac{\pi}{2}$
(30) $\frac{\pi}{2}$

Date: Monday, December 6th, 2010.